Caring for Patients: Beyond Just Complexity

Francesco Giordani
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4th Ortoghonal Weekend - Procida, May 2026

Home Healthcare Routing and Scheduling Problem

Motivations

Why is Home Healthcare relevant?

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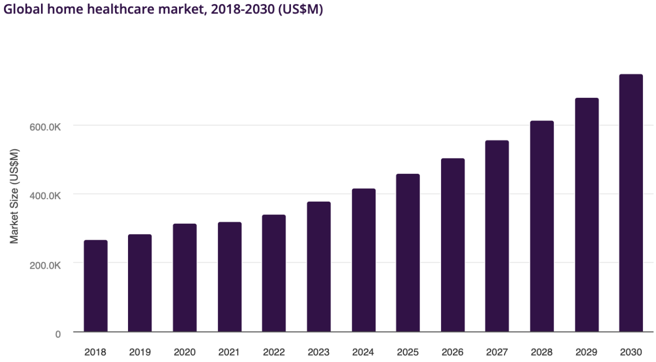
- market proof (after COVID-19)



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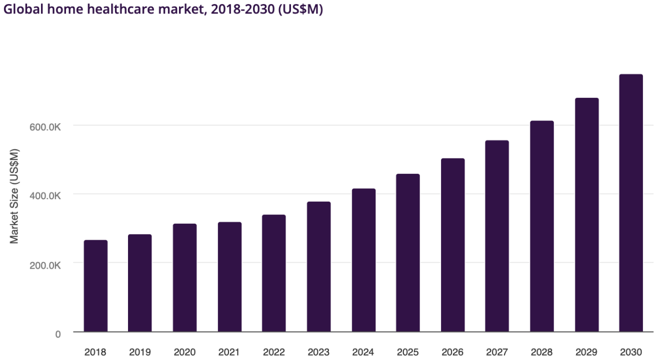
- old people necessities



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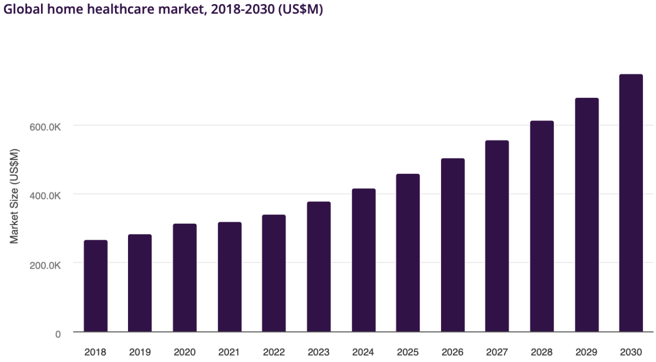
Why is Home Healthcare relevant?

- market proof (after COVID-19)
- old people necessities
- more sustainable



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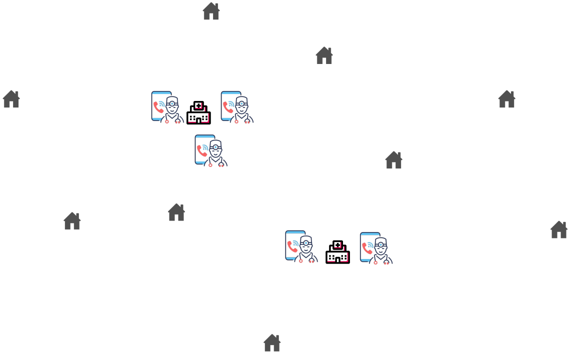

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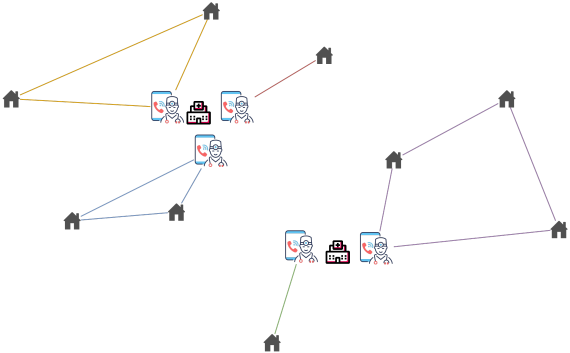

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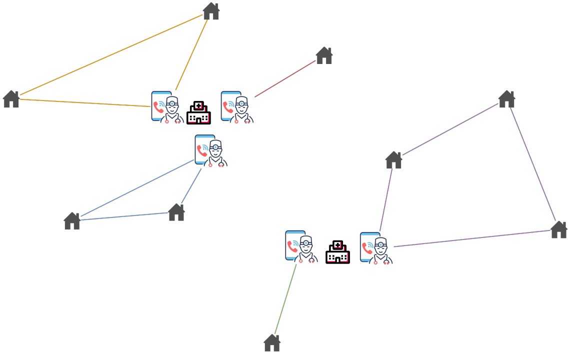

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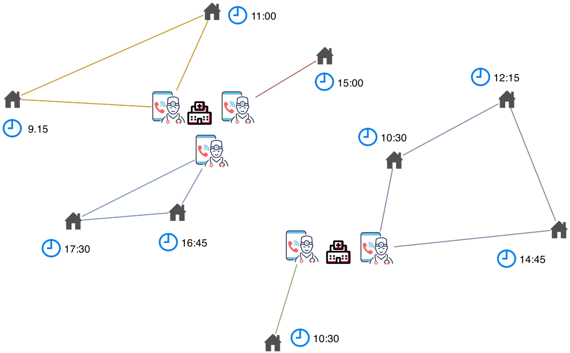

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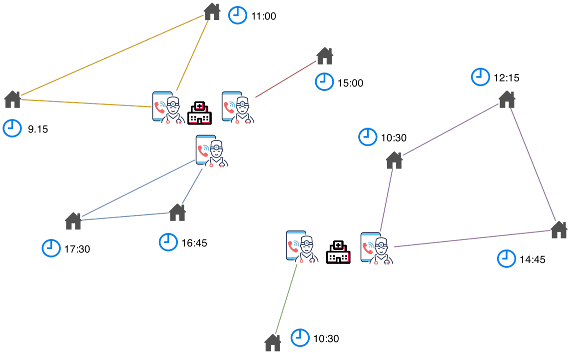

But it's more complex than it seems!

Problem Definition

[Mankowska, Meisel, and Bierwirth, 2014][1]

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- Patients
  - Home location
  - Time window
  - Needed service(s): either one or two
    (two services can be simultaneous or sequential)

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- Objectives
  1. Total travel distance of caregivers
  2. Tardiness at patients (total and highest)

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[Ceschia et al., 2025][2]

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- Caregivers
  - Optional lunch break
- Objectives
  1. Total travel distance of caregivers
  2. Tardiness at patients (total and highest)
  3. Total overtime of caregivers
  4. Total waiting time at patients' locations
  5. Maximum idle time of caregivers
  6. Penalties for patients' preferences
  7. Penalties for caregivers skipping lunch break
  8. Penalties for not visiting optional patients

Impact

Which problems are solved?

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- spend less (time and money)

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- improve quality of care and quality of work

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- spend less (time and money)
- improve quality of care and quality of work
- reduce pollution

Theory of Mathematical Programming

Exact Methods vs Metaheuristics

	Exact Methods	Metaheuristics
Goal	Absolute mathematical perfection	"Good enough, fast enough"
Mechanism	Systematic exploration	Strategic, guided search
Pro	Provides mathematical proof of optimality	Highly scalable; designed to escape local optima
Con	"Combinatorial explosion" (NP-hard)	Cannot guarantee or prove the absolute optimum

Matheuristics [3]

$$\text{Matheuristic} = \text{Mathematical Programming} + \text{Metaheuristics} \tag{1}$$

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Goal: merging the precision of exact methods with the strategic search of heuristics

Simulated Annealing

S. Kirkpatrick; C. D. Gelatt; M. P. Vecchi, 1983 [4]

Solution Representation

Finding a solution fastly:

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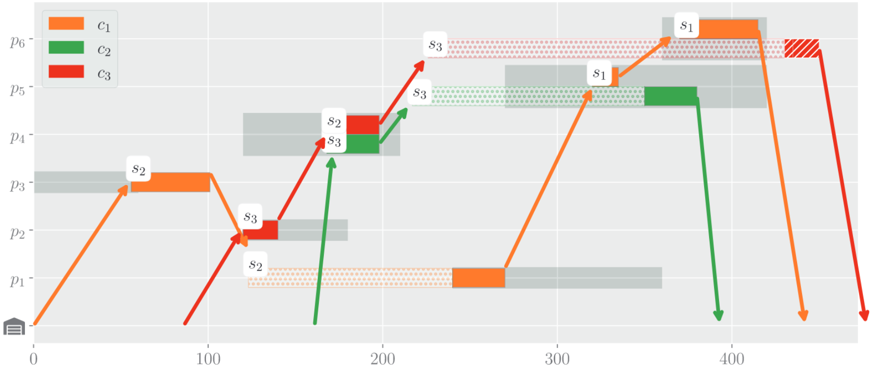
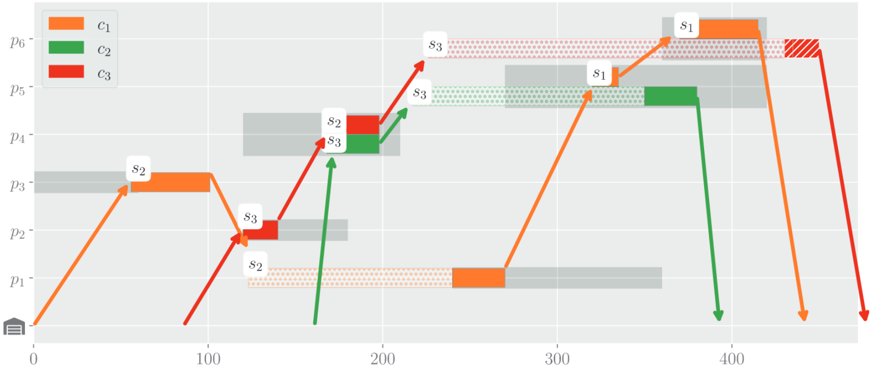
2. Assigned caregivers: $\Theta = [(1,), (3,), (1,), (3, 2), (1, 2), (1, 3)]$



Solution Representation

Finding a solution fastly:

1. Patients global ordering: $\Pi = [3, 2, 4, 1, 5, 6]$
2. Assigned caregivers: $\Theta = [(1,), (3,), (1,), (3, 2), (1, 2), (1, 3)]$
3. Scheduling procedure: "at the earliest" (deterministic)



Initial Solution & Local Neighborhoods

Initial solution

1. random Π
2. random Θ (satisfying abilities)
3. scheduling deterministic procedure

Local Neighborhoods operators

- `MovePatient` $\Pi = [3, 2, 4, 1, 5, 6] \rightarrow [3, 4, 2, 1, 5, 6]$ $\rightarrow$ new solution (local)
- `SwapPatients`
- `InRouteSwap`

High-Level Simulated Annealing


```
Initialize: initial sol s_init, temperatures T_init, T_final, number of iterations I, cooling rate α
while T > T_final do
    for i ∈ I do
        Random neighbor solution s'
        Cost difference: ΔF ← Cost(s') − Cost(s)
        if ΔF ≤ 0 then
            s ← s'
        else
            s ← s' with probability e^(−ΔF/T)
    T ← α · T
return best solution found overall
```

SA: pros & cons

Pros

- Fast
- Simple to implement
- Can escape local minima

Cons

- No guarantee of optimality
- Global search not efficient

Mixed Integer Linear Programming

MILP in a nutshell

Problem definition

$$\begin{aligned} \text{minimize} \quad & c^T x \\ \text{subject to} \quad & Ax \leq b \\ & x_i \in \mathbb{R} \quad \forall i \\ & x_i \in \mathbb{Z} \quad \forall i \in I \end{aligned}$$

MILP for HHRSP [5]

INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

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Intl. Trans. in Op. Res. 0 (2025) 1–47
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A unified formulation for home healthcare routing and scheduling problems

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Abstract

Home Healthcare is an essential component of healthcare systems, where caregivers visit patients' homes to deliver services. While presenting advantages with respect to institutional care, such as being cost-effective and alleviating family burdens, it presents challenges in scheduling and routing caregivers efficiently. While

Objective Function

Minimize

1. Total time distance travelled by the caregivers
2. Total tardiness (delay) in treatments for all patients
3. Maximum tardiness in treatments for all patients
4. Total overtime carried out by the caregivers
5. Total waiting time at patients' locations spent by the caregivers
6. Maximum idle time over all caregivers
7. Total absolute deviation of working time of caregivers with respect to the average working time
8. Total penalty paid due to patients visited by caregivers different from the preferred ones
9. Total penalty accumulated by caregivers who are skipping their lunch breaks
10. Total penalty accumulated by not visiting optional patients

Objective Function

$$
\begin{aligned}
\min \lambda_1 \left(\sum_{i \in C_0'} \sum_{j \in C_0, j \neq i} d_{ij} \sum_{v \in V_i \cap V_j} x_{ij}^v + \sum_{v \in V_{lb}} b_v \right) + \lambda_2 \sum_{i \in C} z_i \\
+ \lambda_3 D_{max} + \lambda_4 \sum_{v \in V} y_v + \lambda_5 \sum_{i \in C} w_i + \lambda_6 H_{max} + \lambda_7 \sum_{v \in V} s_v \\
+ \lambda_8 \sum_{i \in C} \sum_{v \in V, v \notin P_i} (1 - x_{ii}^v) + \lambda_9 \sum_{v \in V_{lb}} x_{c_v^{lb} c_v^{lb}}^v + \lambda_{10} \sum_{i \in C_{opt}} v_i
\end{aligned} \tag{2}
$$

Routes Variables

$$x_{ij}^{v} = \begin{cases} 1 & \text{if caregiver } v \text{ travels from patient } i \text{ to patient } j \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

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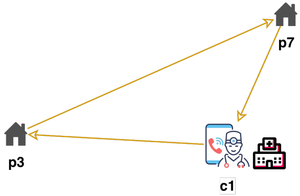

$$x_{0,3}^{1} = 1 \qquad x_{3,7}^{1} = 1 \qquad x_{7,0}^{1} = 1$$

A routing constraint

For each caregiver v and each location i, exactly one x variable must be active (partitioning constraint)

$$\sum_{j \in C_0^v} x_{ji}^v = 1 \qquad v \in V, i \in C_0^v, \tag{4}$$

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However, the complete model has

- 19 sets
- 12 parameters
- 12 variables
- 20 constraints
- too many (read the paper if you want)

MILP: pros & cons

Pros

- Optimality guarantee

Cons

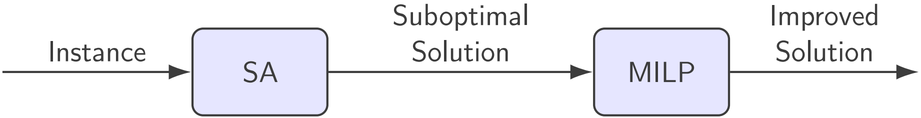
- Can be very slow for large instances

MILP Model Results

	Features					SA	MILP	
ID	Patients	Caregivers	Services	Double Service Patients	Optional Patients	UB	UB	t [s]
i-054	200	30	220	10%	10%	**65188**	–	–
i-077	100	17	120	20%	20%	**16164**	–	–
i-083	70	11	98	40%	0%	**12831**	–	–
i-100	25	5	35	40%	0%	15119	**14722**	3600
i-116	10	4	13	30%	60%	17393	**16973**	1.0
i-126	200	32	260	30%	20%	**47475**	–	–
i-134	15	3	17	13%	60%	15820	**15616**	1000
i-164	200	27	240	20%	0%	**58873**	–	–
i-167	100	13	109	9%	100%	**13805**	–	–
i-175	450	63	539	20%	0%	**34014**	–	–
i-185	340	62	442	30%	20%	**26591**	–	–
i-219	100	18	130	30%	20%	**11086**	–	–
i-235	25	6	44	76%	48%	**6021**	7242	3600
i-247	25	5	32	28%	48%	21186	**19507**	3600
i-250	190	26	217	14%	30%	**28639**	–	–
i-263	250	42	324	30%	20%	**36933**	–	–
i-272	170	26	194	14%	50%	**27672**	–	–
i-316	25	5	50	100%	0%	**10196**	10270	3600
i-360	200	35	300	50%	20%	**46205**	–	–
i-369	75	13	97	29%	20%	**25225**	–	–
i-406	100	18	130	30%	50%	**25260**	–	–
i-414	70	11	98	40%	0%	**33457**	–	–
i-416	400	46	400	0%	0%	**41009**	–	–
i-446	75	13	111	48%	20%	**21463**	–	–

Best of both worlds?

Warm Start using SA



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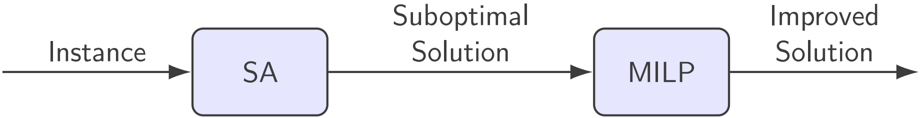
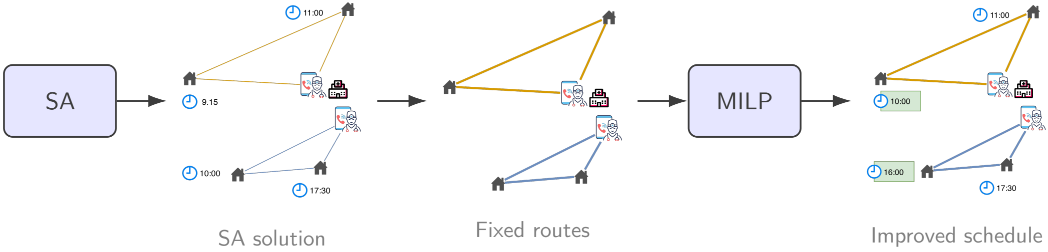


2 approaches:

- **Hard** Warm Start
- **Hint** Warm Start

Fix x^{v}_{ij} using SA solution, let MILP optimize scheduling



Hard Warm Start: pros & cons

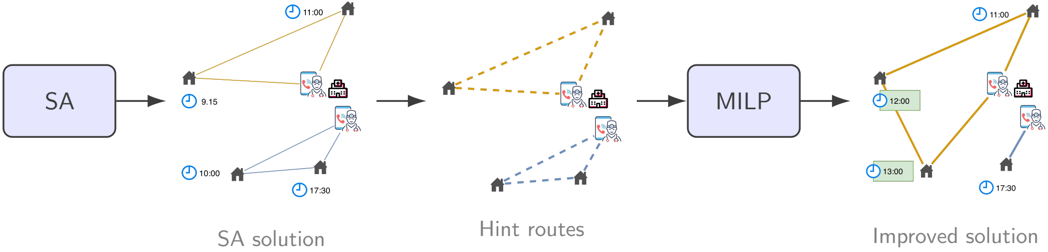
Pros

- Fast

Cons

- Less flexibility (improving only schedule, suboptimal)

Initialize x_{ij}^{v} using SA solution, let MILP optimize both routing and scheduling



Hint Warm Start: pros & cons

Pros

- Optimality guarantee (if MILP converges)

Cons

- Faster than MILP alone, but still slow

Warm Start Results

	SA	MILP		Hard WS		Hint WS	
ID	UB	UB	t [s]	UB	t [s]	UB	t [s]
i-054	65188	–	–	64543	1.16	**64026**	3600
i-077	16164	–	–	15834	1.08	**15546**	3600
i-083	12831	–	–	12655	0.51	**12384**	3600
i-100	15119	**14722**	3600	15053	0.04	14887	3600
i-116	17393	**16973**	1.0	17393	0.00	**16973**	1
i-126	47475	–	–	**47024**	3600.21	**47024**	3603
i-134	15820	**15616**	1000	15739	0.00	**15616**	3600
i-164	58873	–	–	**58111**	5.55	–	–
i-167	13805	–	–	**13786**	0.78	**13786**	3600
i-175	34014	–	–	**33994**	116.73	–	–
i-185	26591	–	–	**26441**	64.93	–	–
i-219	11086	–	–	11212	1.24	**10479**	3600
i-235	6021	7242	3600	5985	0.09	**5924**	3600
i-247	21186	**19507**	3600	21186	0.03	19631	3600
i-250	28639	–	–	**28609**	4.66	**28609**	3600
i-263	36933	–	–	**36705**	16.99	**36705**	3600
i-272	27672	–	–	**27602**	4.03	–	–
i-316	10196	10270	3600	10170	0.09	**10131**	3600
i-360	46205	–	–	**45882**	1234.28	**45882**	3600
i-369	25225	–	–	25128	0.48	**24364**	3600
i-406	25260	–	–	25179	1.50	**24922**	3600
i-414	33457	–	–	33222	0.54	**33016**	3600
i-416	**41009**	–	–	**41009**	28.61	–	–
i-446	21463	–	–	21322	0.78	**20693**	3600

Large Neighborhood Search

Shaw, 1998 [6]

LNS Algorithm

Core Idea: iteratively destroy (using a global neighborhood) and repair the solution [7]
Goal: improving global search

```
1: s ← Initial Solution
2: while stopping criterion not met do
3:     s_new ← Repair(Destroy(s))
4:     if Cost(s_new) ≤ Cost(s) then            ▷ Accept new sol (greedy)
5:         s ← s_new
6: return best solution found overall
```

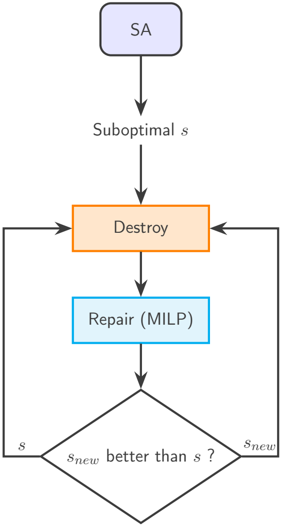
Destroy & Repair Operators

Destroy operators ($p \in [0, 1]$ hyperparameter)

1. Destroy $p\%$ routes randomly
2. Destroy $p\%$ routes baricenter-based

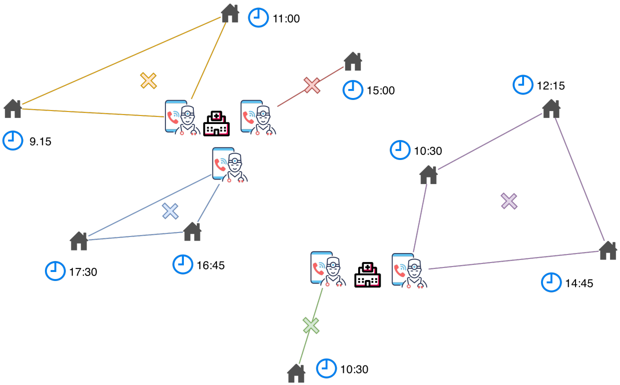
Repair operator

- MILP solver with time limit



Baricenter-based Destroyer

Idea: given a caregiver v, destroy its route and the baricenter-closest ones



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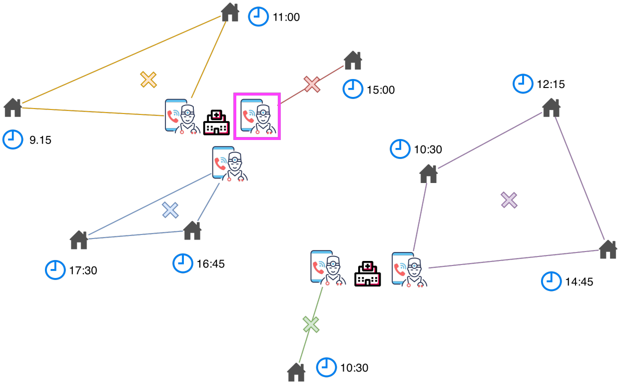

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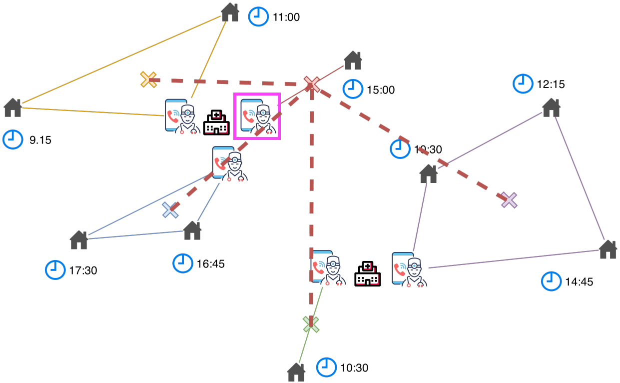

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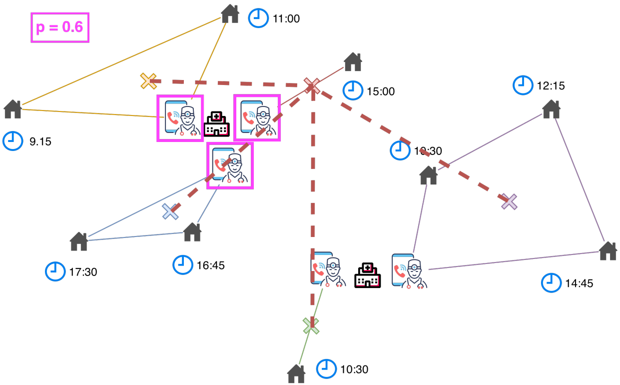

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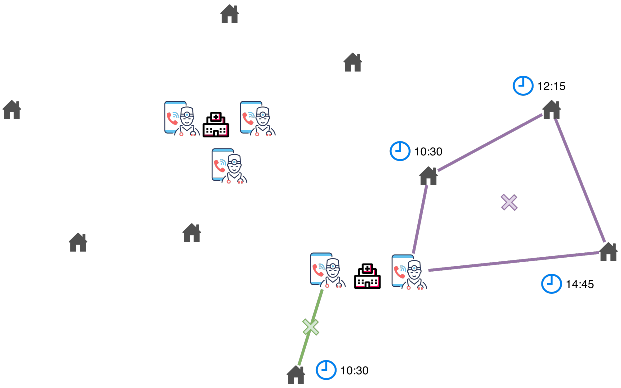

LNS: pros & cons

Pros

- Good balance between exploration and exploitation

Cons

- Destroyer could be biased, leading to suboptimal solutions

LNS Results

	SA	MILP		Hard WS		Hint WS		rnd LNS		baricenter LNS	
ID	UB	UB	t [s]	UB	t [s]	UB	t [s]	UB	t [s]	UB	t [s]
i-054	65188	–	–	64543	1.16	64026	3601	61847	3600	**61399**	3600
i-077	16164	–	–	15834	1.08	15546	3600	**14987**	3600	15253	3600
i-083	12831	–	–	12655	0.51	12384	3600	**12128**	3600	12371	3600
i-100	15119	**14722**	3600	15053	0.04	14887	3600	14807	3600	14882	3600
i-116	17393	**16973**	1.0	17393	0.00	**16973**	1	**16973**	3600	**16973**	3600
i-126	47475	–	–	47024	3600.21	47024	3603	45717	3600	**45616**	3600
i-134	15820	**15616**	1000	15739	0.00	**15616**	3600	**15616**	3600	**15616**	3600
i-164	58873	–	–	58111	5.55	–	–	**55347**	3600	55806	3600
i-167	13805	–	–	13786	0.78	13786	3600	13747	3600	**13392**	3600
i-175	34014	–	–	33994	116.73	–	–	33994	3600	**33786***	3600
i-185	26591	–	–	26441	64.93	–	–	26441	3600	**26238**	3600
i-219	11086	–	–	11212	1.24	10479	3600	**10276**	3600	10552	3600
i-235	6021	7242	3600	5985	0.09	5924	3600	**4122**	3600	4384	3600
i-247	21186	**19507**	3600	21186	0.03	19631	3600	**19507**	3600	19630	3600
i-250	28639	–	–	28609	4.66	28609	3610	28405	3600	**28243**	3600
i-263	36933	–	–	36705	16.99	36705	3606	35740	3600	**35476**	3600
i-272	27672	–	–	27602	4.03	–	–	27471	3600	**27449**	3600
i-316	10196	10270	3600	10170	0.09	**10131**	3600	**10131**	3600	**10131**	3600
i-360	46205	–	–	45882	1234.28	45882	3605	**44203**	3600	45090	3600
i-369	25225	–	–	25128	0.48	24364	3600	**23802**	3600	24743	3600
i-406	25260	–	–	25179	1.50	24922	3600	**24783**	3600	24897	3600
i-414	33457	–	–	33222	0.54	33016	3604	32134	3600	**31979**	3600
i-416	41009	–	–	41009	28.61	–	–	40991	3600	**40847**	3600
i-446	21463	–	–	21322	0.78	20693	3600	**19970**	3600	20833	3600

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